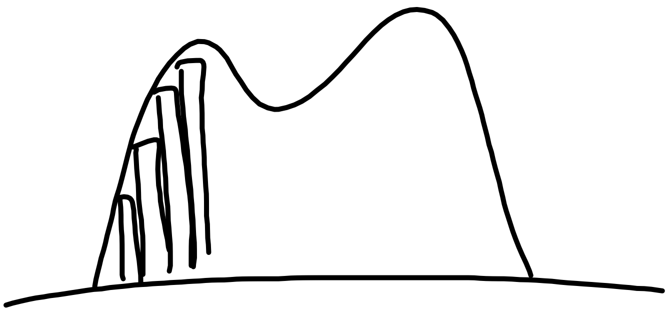
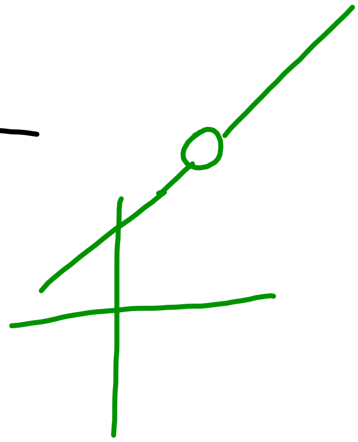
Unit Outline:

Limit Definitions & Notation Well-Behaved Functions Where Limit Does Not Exist	11.1
Not-So-Well-Behaved Functions	11.2
Difference Quotient Tangent Line	11.3
Derivative Limits at Infinity	11.4
Limit of Sequence Area Problem	11.5

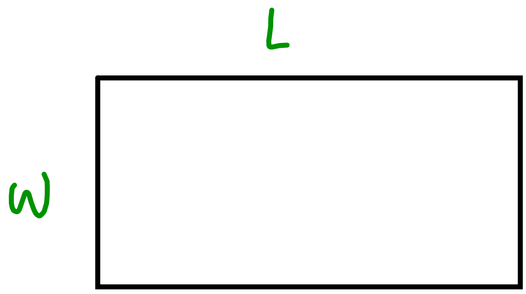




You have 24 inches of wire. Make a rectangle that has the largest area possible.

$2w + 2L = 24$

W	L
2	10
2.999	10.



What is a limit?

something that bounds, restrains, or confines

a number whose numerical difference from a mathematical function is arbitrarily small for all values of the independent variables that are sufficiently close to but not equal to given prescribed numbers or that are sufficiently large positively or negatively

http://www.merriam-webster.com/dictionary/limit

Definition 1 Let *f(x)* be a function defined on an interval that contains $x = a$, except possibly at $x = a$. Then we say that,

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is some number $\delta > 0$ such that

$$|f(x) - L| < \varepsilon \qquad \text{whenever} \qquad 0 < |x - a| < \delta$$

http://tutorial.math.lamar.edu/Classes/CalcI/DefnOfLimit.aspx

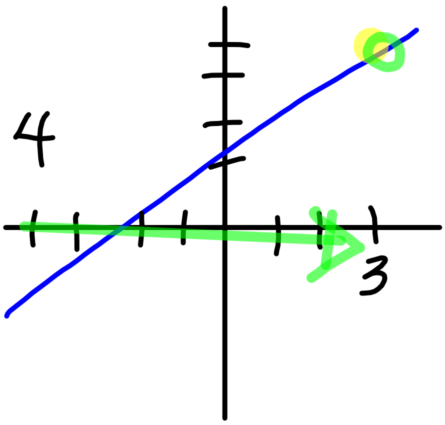
Math Terminology

Write: $\lim_{x \to c} f(x) = L$

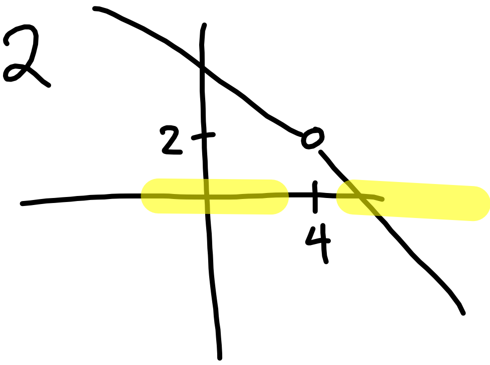
Read: "The limit of f of x, as x approaches c, equals L."

But What Does it Mean?

$\lim_{x \to 3} x+1 = 4$



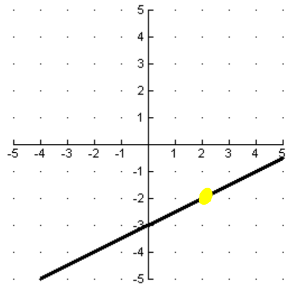
$\lim_{x \to 4}$ — " " $= 2$



$$f(x)=\frac{1}{2}x-3$$

x	1.97	1.98	1.99	2.00	2.01	2.02	2.03
f(x)	-2.015	-2.01	-2.005	ø	-1.995	-1.99	-1.985

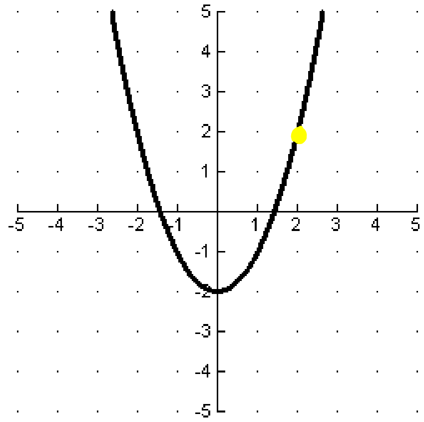
$$\lim_{x\to 2}\left(\frac{1}{2}x-3\right)= -2$$



$$f(x) = x^2 - 2$$

x	1.97	1.98	1.99	2.00	2.01	2.02	2.03
f(x)							

$$\lim_{x \to 2} \left(x^2 - 2 \right) = 2$$



So, what does a limit mean to us?

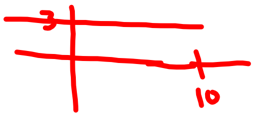
Try some on your own

Find a partner

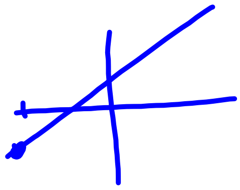
Work problem, exchange paper, compare

For each limit, set up table and graph in calculator w/ y for f(x)

$$\lim_{x\to 10} 3 = 3$$



$$\lim_{x\to -8}(x+4) = -4$$



$$\lim_{x\to \pi}(\sin x) = 0$$

What did you notice about these limits?

no domain issues

For Well behaved functions,
the limit is found by evaluating the function.
This is called Direct Substitution.

What types of functions are well-behaved?

Constant
Linear
Polynomial
Sine/Cosine
Rational (if denominator $\neq$ 0)
Radical (if radicand $\geq$ 0)

$$\frac{x}{x^2+1}$$

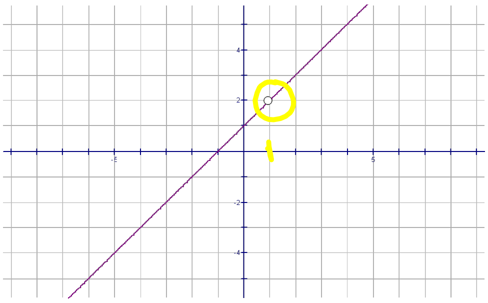
$$\sqrt{x^2+1}$$

But what about this one?

$$f(x)=\frac{x^2-1}{x-1}=\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

x	.9	.95	.99	1.00	1.05	1.1	1.15
f(x)	1.9	1.95	1.99	$\varnothing$	2.05	2.1	2.15

$$\lim_{x\to 1} f(x)=2$$



Check these with same partner:

1) Are they well behaved at the input limit?
2) Is there a limit value there?
hint: graph fn, table set at input value, change ∆Tbl

$$\lim_{x \to -3}\left(\frac{\sqrt{1-x}-2}{x+3}\right) =$$

$$\lim_{x \to 0}\left(\frac{\cos x - 1}{x}\right) =$$

A limit can exist, even if the function does not have
a value at that point.

However, you must be able to reach this limit
coming from both directions.

Still, not all function have limits.

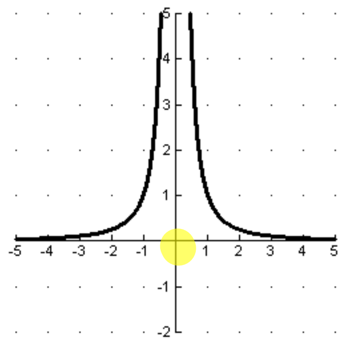
Three cases where as limit does not exist (DNE)

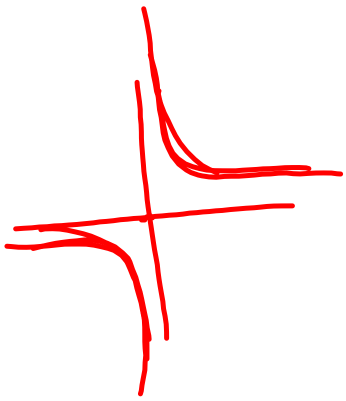
1. Unbounded behavior
2. Does not settle to a single value (oscillates)
3. Has different left and right side limits

Unbounded behavior

$$\lim_{x\to 0}\left(\frac{1}{x^2}\right)$$

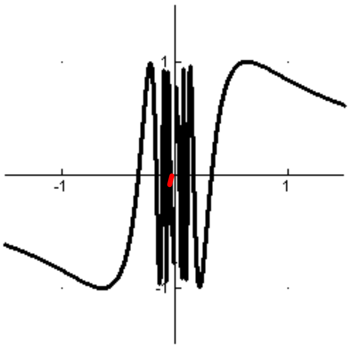
DNE





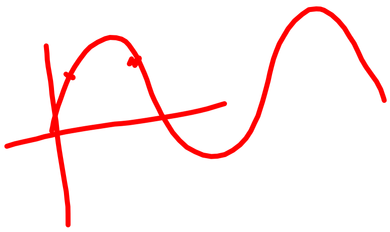
Oscillating behavior

$$\lim_{x \to 0}\left[\sin\left(\frac{1}{x}\right)\right]$$



sin 1 over x.ggb

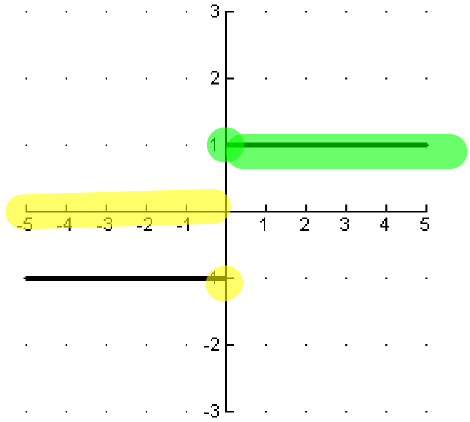
DNE

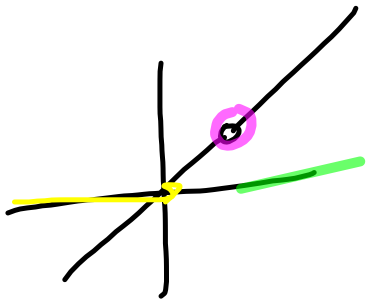


x	$\frac{1}{x}$	$\sin\left(\frac{1}{x}\right)$
$\frac{2}{\pi} \approx 0.637$	$\frac{\pi}{2}$	1
$\frac{1}{\pi} \approx 0.318$	π	0
$\frac{2}{3\pi} \approx 0.212$	$\frac{3\pi}{2}$	-1
$\frac{1}{2\pi} \approx 0.159$	2π	0
$\frac{2}{5\pi} \approx 0.127$	$\frac{5\pi}{2}$	1
$\frac{1}{3\pi} \approx 0.106$	3π	0
$\frac{2}{7\pi} \approx 0.091$	$\frac{7\pi}{2}$	-1
$\frac{1}{4\pi} \approx 0.080$	4π	0

Different Left and Right Behavior

$$\lim_{x \to 0} \frac{|x|}{x}$$ DNE



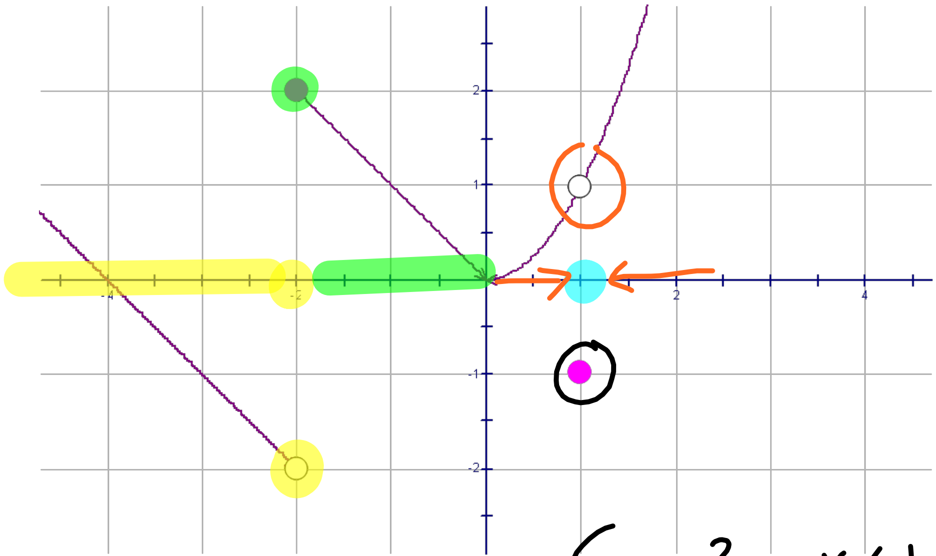


Tale of Two Limits

$$\lim_{x \to -2} f(x) =$$

DNE

$$\lim_{x \to 1} f(x) = 1$$



$$\begin{cases} x^2, & x < 1 \\ -1, & x = 1 \\ x^2, & x > 1 \end{cases}$$

So far: Numerical, Graphical, Direct Substitution

Fine point #1: $$\lim_{x\to 0}(1+x)^{\frac{1}{x}}$$

(1+x)^(1/x) graph

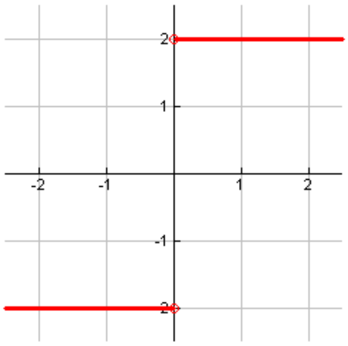
average points
using table

$$\lim_{x\to 0}\frac{\sin x}{x}$$

sinx/x graph

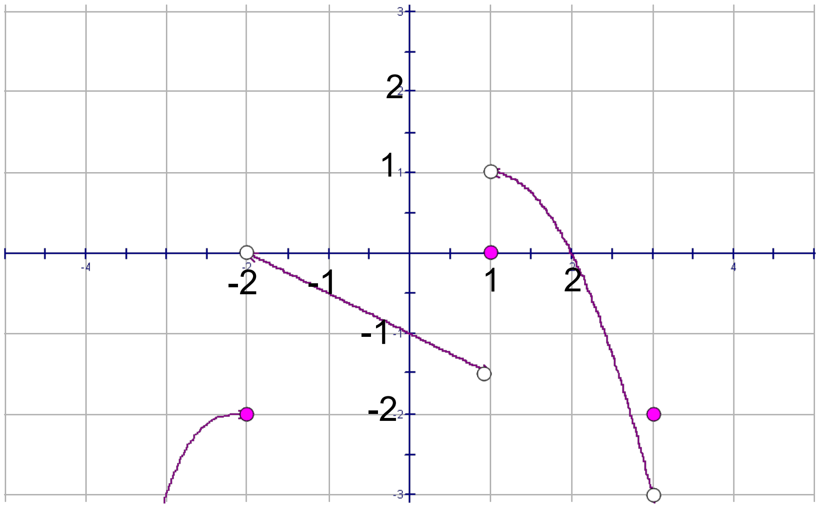
Fine point #2:

left side,
right side limits



$$\lim_{x \to 0^-} \frac{|2x|}{x} \qquad \lim_{x \to 0^+} \frac{|2x|}{x}$$

Note the + and - signs!

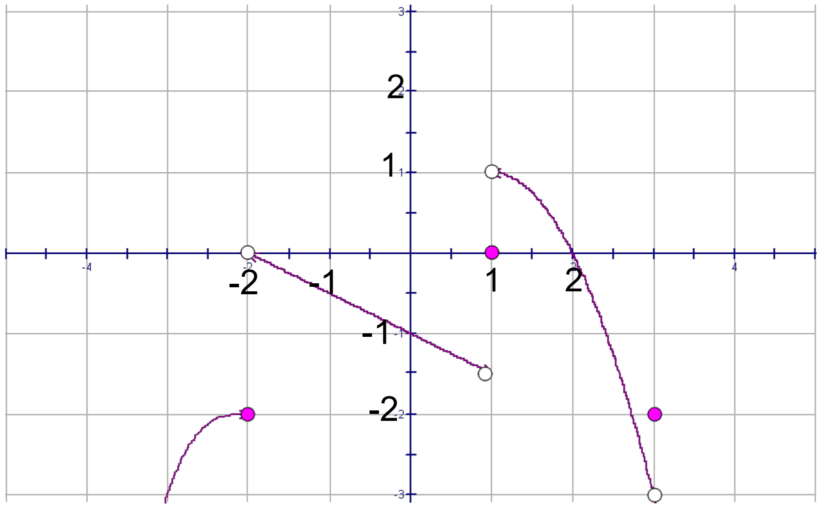


$f(-2) =$

$\lim_{x \to -2^-} f(x) =$

$\lim_{x \to -2^+} f(x) =$

$\lim_{x \to -2} f(x) =$



$f(1) =$

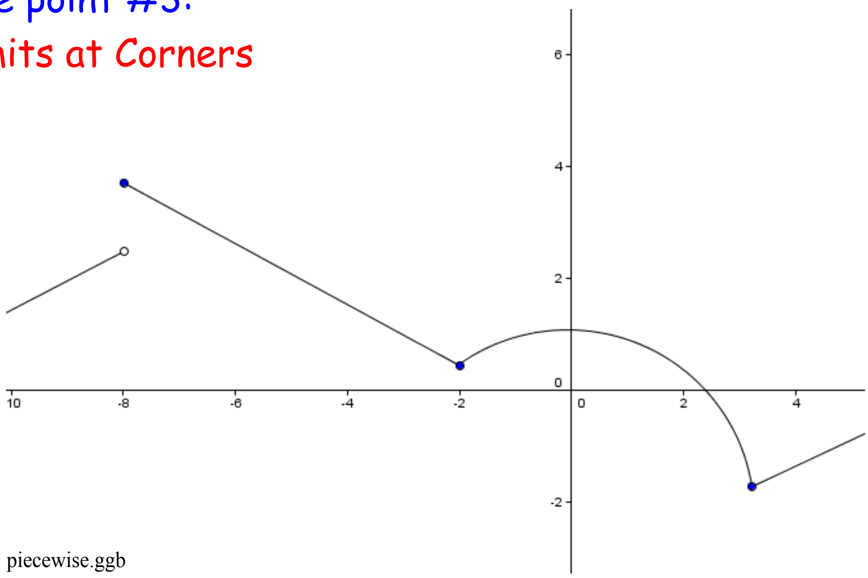
$\lim_{x \to 1^-} f(x) =$

$\lim_{x \to 1^+} f(x) =$

$\lim_{x \to 1} f(x) =$

Fine point #3:

Limits at Corners



piecewise.ggb

More practice?

$$\lim_{x \to 3}\left(\frac{7}{x-3}\right) =$$

$$\lim_{x \to 1}\left(\sin \pi x\right) =$$

$$\lim_{x \to 3}\left(\frac{x^2+1}{x}\right) =$$

$$\lim_{x \to 8}\left(\frac{\sqrt{x+1}}{x-4}\right) =$$

Other Techniques

Properties of Limits

1. $\lim_{x \to c} b = b$

2. $\lim_{x \to c} x = c$

3. $\lim_{x \to c} x^n = c^n$

4. $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$

examples -- f(x) = 3x + 2   g(x) = x^2 -1

$$1.\ \lim_{x\to c} bf(x) = b\lim_{x\to c} f(x)$$

$$2.\ \lim_{x\to c}\left[f(x) \pm g(x)\right] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$$

$$3.\ \lim_{x\to c}\left[f(x)g(x)\right] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right]$$

$$4.\ \lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)},\ \text{if } \lim_{x\to c} g(x) \neq 0$$

$$5.\ \lim_{x\to c}\left[f(x)\right]^n = \left[\lim_{x\to c} f(x)\right]^n$$

1+x ^ 1overx.ggb

sin x over x.ggb

piecewise.ggb

sin 1 over x.ggb